

Quantum random walk : effect of quenching

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We study the effect of quenching on a discrete quantum random walk by removing a detector placed at a position x_D abruptly at time t_R from its path. The results show that this may lead to an enhancement of the occurrence probability at x_D provided the time of removal $t_R < t_R^{lim}$ where t_R^{lim} scales as x_D^2 . The ratio of the occurrence probabilities for a quenched walker ($t_R \neq 0$) and free walker ($t_R = 0$) shows that it scales as $1/t_R$ at large values of t_R independent of x_D . On the other hand if t_R is fixed this ratio varies as x_D^2 for small x_D . The results are compared to the classical case. We also calculate the correlations as functions of both time and position.

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I. INTRODUCTION

Quenching phenomena has been a much studied topic in both classical and quantum systems in the recent past. A quenching process can be broadly defined as one in which a certain quantity or condition of the system (e.g., temperature, magnetic field etc.) is changed in time from an initial value to a final value in a certain manner. Quenching can be studied in several possible ways. In certain cases, slow quenching is applied to obtain the classical ground states of the system as in the case of a spin glass system where there are many minima in the energy landscape, separated by barriers which maybe overcome by quantum tunnelling [1, 2]. On the other hand, in systems with a quantum critical point, nonequilibrium dynamics is studied by fast or slow quenching of relevant variables (usually a field or interaction which is made time dependent). Examples include transverse Ising and XY models [2–5]. In ultracold atoms in an optical lattice fast quenching is applied by shifting the position of the trap potential and studying its response [6, 7]. In case of the transverse Ising or XY model, there is a deviation from the equilibrium state under quenching as the quantum critical point is crossed and the quantity of interest is the ‘defect’, which is the amount of departure from the actual equilibrium value.

We consider here a discrete quantum random walk (QRW) [8–11], completely different from a classical random walk [12–14], on which the effect of quenching is studied. The QRW is made unitary by coupling the translation with chirality or rotation. The state of the walker is expressed in the $|x\rangle|d\rangle$ basis, where $|x\rangle$ is the position (in real space) eigenstate and $|d\rangle$ is the chirality eigen state (either left or right).

II. QUENCHED QUANTUM WALK : SCHEME AND MEASUREMENTS

Usually, a quantum random walk is studied with two kinds of boundary conditions. In the infinite walk (IW), there is no boundary and in the semi-infinite walk (SIW) there is one absorbing boundary. Measurement-wise this signifies that there is no detector in the first case up to the time of observation, while there is a detector all the time in a particular position in the second.

We consider the case when a detector initially placed at x_D is removed suddenly at time t_R . This is termed a quenching phenomena as the presence of the detector and its subsequent removal may be compared to the presence of a time dependent transverse field in Ising model, XY model etc. We call this walk the quenched quantum walk (QQW).

Classically, even when there is an absorbing boundary, a random walker will simply be either completely absorbed there, or if not, will propagate like a free walker. In the quantum case on the other hand, the walk exists with non-zero probability at different locations and the absorption will thus take place with a probability. Experimentally, this means that if the particle is detected, the evolution will be stopped. But if not, the walker will survive with non-zero and modified probabilities at the other locations [15] which is completely different from one which propagates without a detector [16].

In the QRW, the position of the particle, $\psi(x, t)$ is given as,

$$\psi(x, t) = \begin{bmatrix} \psi_L(x, t) \\ \psi_R(x, t) \end{bmatrix}. \quad (1)$$

Here we have chosen the Hadamard coin [9, 17] unitary operator H to perform the rotation which is coupled to the translation. H is given by

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (2)$$

The occupation probability of site x at time t is given by $|\psi_L(x, t)|^2 + |\psi_R(x, t)|^2$; sum of these probabilities over all x is 1. The walk is initialized at the origin with

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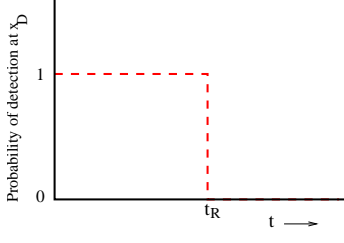


FIG. 1: (Color online) The probability of the walker getting detected at a point x_D is shown against time.

$\psi_L(0,0) = a_0, \psi_R(0,0) = b_0; a_0^2 + b_0^2 = 1$. We have taken a symmetric walk; $a_0 = 1/\sqrt{2}, b_0 = i/\sqrt{2}$.

Let the detector be placed at some given site x_D . In general, if the particle is at x_D with probability α and the detector detects the particle with probability β , then the total absorption probability at x_D will be $\alpha\beta$. In our case we have chosen $\beta = 1$. Removing the detector at time t_R is equivalent to having a step function behaviour of the probability of being detected at x_D as a function of time as shown in Fig. 1.

At each time step when the ensemble is measured the amplitudes describe only to the surviving copies. We may choose our scheme of measurement in two ways. Let the normalised occupation probability at x at time t be denoted by $\tilde{f}(x,t)$. Thus $\tilde{f}(x,t)$ denotes the fraction of the copies that survived the measurement up to time t_R (not the fraction of the initial population) which reaches x at time t . If the particle survives absorption, then $\tilde{f}(x,t)$ will be the conditional probability of finding it at x at time t in a ‘single’ observation. However, the average measure $f(x,t)$ takes into account the absorption probability and is given by $\tilde{f}(x,t) \times$ survival probability; $\sum_x \tilde{f}(x,t) = 1$ whereas $\sum_x f(x,t) = 1 - d$ where d is the probability that it was absorbed earlier. This issue of two types of measurement was already addressed in [16].

As x_D and t_R are the parameters of the system, we further modify our notation: $\tilde{f}(x,t,x_D,t_R)$ and $f(x,t,x_D,t_R)$ are the normalised and average occupation probability of site x at time t respectively given x_D and t_R .

Certain limiting cases can be immediately identified:

(i) If $t_R = 0$ then the normalised and average occupation probabilities become same and are identical to the usual occupation probability of a quantum random walker when there is no detector at all; the IW case.

(ii) If $t_R = \infty$, it is a case of SIW.

(iii) If $x_D \gg 0$, the walk will be IW-like in finite times.

The study for quench must be for a time $t \geq t_R \geq x_D$, otherwise the removal of the detector does not affect the probabilities. [Note that, here the numerical value of x_D is considered while comparing with time t .]

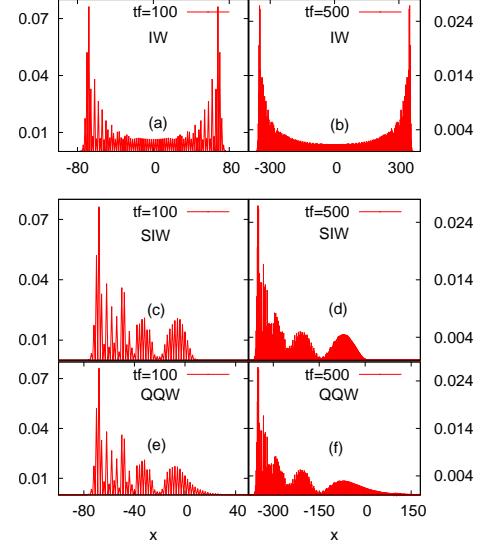


FIG. 2: (Color online) Snapshots of the probabilities of occupation f for a symmetric walker under different conditions are shown for $t = 100$ (left panel) and 500 (right panel) against position x . (a) and (b) are results for IW. In (c) and (d) the results are shown for a SIW when a detector is placed at $x_D = 10$ for all times. In (e) and (f) QQW results are shown where the detector is initially placed at $x_D = 10$ but is removed at time $t_R = 50$.

III. RESULTS

We first present some snapshots of the probabilities of occupation $f(x)$ of different sites at different times for the IW, SIW and QQW in Fig. 2 to get a comparative picture of the dynamics in the three cases. For a quantum walker, the displacement in time t is proportional to t and the maximum of the occupation probability occurs at a value of $x \sim t/\sqrt{2}$. These features are clearly shown for the IW. For the SIW and QQW, the detector placed at x_D will make the probabilities different; up to $t = t_R$, QQW and SIW are equivalent. However, for $t > t_R$, as shown in the figure, the probability “spills out” beyond x_D for the QQW, although far away from x_D , there is little difference. In fact away from the boundary, we find that maximum probability is again at a value of $|x| \sim t/\sqrt{2}$.

We next present the data for a fixed value of $x = x_D$ at different times t for both $\tilde{f}(x,t,x_D,t_R)$ and $f(x,t,x_D,t_R)$, using the shorthand notation \tilde{f} and f for these two quantities respectively. To study the effect of quenching, the ratios of the probabilities at different positions and times for the IW and the QQW may be calculated for given values of x_D and t_R . This is in tune with the measure of defects in studies of quenching in the quantum spin models. The ratios \tilde{f}/f_0 and f/f_0 are computed where f_0 is the occupation probability for the IW. \tilde{f}/f_0 shows that it can attain values much larger than 1 for a short time $t > t_R$ before saturating at larger times to values larger or smaller than 1 depending on t_R .

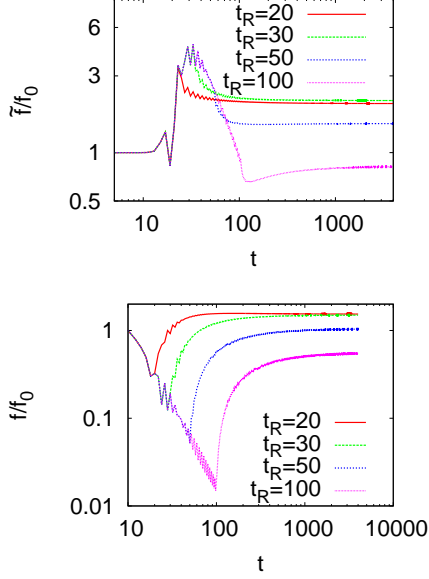


FIG. 3: (Color online) Top: Ratio of $\tilde{f}(x_D, t, x_D, t_R)$ and f_0 for $x_D = 10$ and $t_R = 20, 30, 50, 100$. Bottom: Ratio of $f(x_D, t, x_D, t_R)$ and f_0 for $x_D = 10$ for the same values of x_D and t_R .

These are shown in Fig. 3.

The ratio of f/f_0 are also clearly different from 1 beyond $t = t_R$. However, it shows a much more regular behaviour compared to \tilde{f}/f_0 although it takes a longer time to saturate (Fig. 3). As the detector is placed at x_D , up to $t = x_D$, the walker is not affected by the presence of the detector and the ratios are equal to 1 for all t_R . Beyond $x_D < t < t_R$, the initial population of the ensemble decreases due to detection at site x_D . As time goes on, larger fraction of the initial population get lost and the ratio thus decreases gradually. Then as the detector is removed, the probability grows at x_D , such that the ratio starts increasing before reaching the saturation value. In fact the results are of interest mainly at $t > t_R$, after the detector is removed, when the QQW is different from the SIW and the ratio starts growing. Interestingly, contrary to naive expectation, the ratio may saturate at values larger than unity for small t_R . The explanation for $(f/f_0)_{sat}$ having value > 1 is given later in the paper after we present the results for $x \neq x_D$. The saturation values of the ratio, $(f/f_0)_{sat}$ plotted against t_R shows an initial non-monotonic behaviour, but for large t_R , $(f/f_0)_{sat}$ clearly scales as $1/t_R$ for all values of x_D (Fig. 4).

From Fig. 4, we note that there is a value of t_R beyond which $(f/f_0)_{sat}$ is never > 1 , i.e., beyond this particular $t_R = t_R^{lim}$, $(f/f_0)_{sat}$ monotonically decreases below 1. We find that t_R^{lim} varies as x_D^2 (Fig. 5). Moreover, for fixed t_R , the saturation value of the ratios show a variation with x_D : for large t_R , $(f/f_0)_{sat}$ varies as x_D^2 , for smaller values of t_R , this variation is valid over a small range of x_D . Combining the above two results, we con-

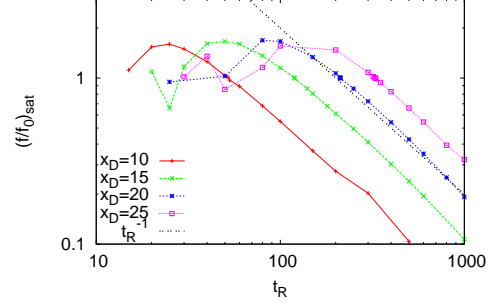


FIG. 4: (Color online) Plots of $(f/f_0)_{sat}$ against t_R for $x_D = 10, 15, 20, 25$ respectively. For large t_R the log-log plot shows a slope of -1 .

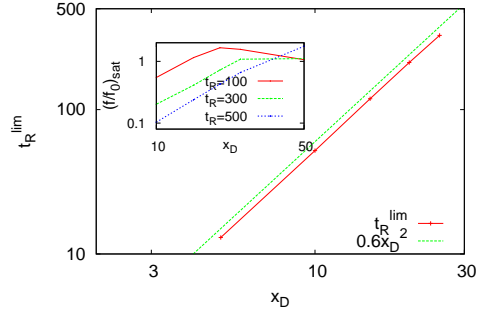


FIG. 5: (Color online) Plot of t_R^{lim} against x_D . The log-log plot shows a slope of 2. In the inset $(f/f_0)_{sat}$ against x_D is plotted for different t_R .

clude, for large t_R ,

$$(f/f_0)_{sat} = kx_D^2/t_R \quad (3)$$

where k is a constant with dimension of inverse length. That $(f/f_0)_{sat}$ should decrease with t_R is expected as the probability of detection increases with larger t_R . On the other hand as x_D is increased, the walk remains unaffected for longer times which implies that $(f/f_0)_{sat}$ should increase with x_D ($x_D \rightarrow \infty$ makes $f = f_0$). However, the exact scaling form eq. (3) is not obvious.

Before discussing other issues it is worthwhile to compare the results of the quantum case to the classical quenched random walk (CQW) under identical fast quenching. For a classical random walker, the occupation probability $f_c(x, t)$ is simply given by $f_{c0}(x, t) \times \text{survival probability}$, where $f_{c0}(x, t)$ is the occupation probability for the classical walker in absence of any boundary. The survival probability is given by $1 - \int_0^{t_R} F_c(x_D, t) dt$, where $F_c(x_D, t)$ is the first passage probability at x_D at time t . So for the classical case,

$$\frac{f_c}{f_{c0}} = 1 - \int_0^{t_R} F_c(x_D, t) dt, \quad (4)$$

and is always less than 1. Moreover, it is independent of x and t . f_c/f_{c0} for $t_R \rightarrow \infty$ scales as $\frac{x_D}{\sqrt{t_R}}$ [14]. For the

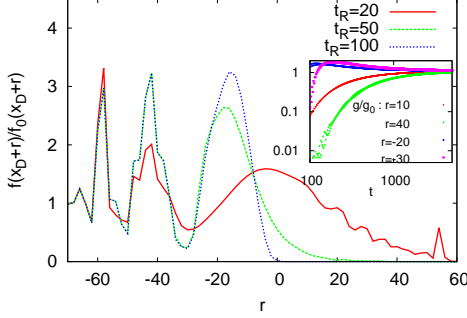


FIG. 6: (Color online) Ratio of the occupation probabilities $f(x_D + r)/f_0(x_D + r)$ against r for $x_D = 10$ and $t_R = 20, 50, 100$ at a particular time $t = 100$. Inset shows g/g_0 against t for $t_R = 50$ for different r .

quantum case in contrast $(f/f_0)_{sat}$ has a different scaling behaviour with $x_D/\sqrt{t_R}$ as already shown.

In fact for a CQW, the ratio is simply identical to the persistence probability at time t_R , i.e., the probability that the walker has not visited the site x_D till time t_R . For the quantum walker, the corresponding persistence probability is proportional to $\frac{1}{t_R^2}$ [16]. Hence the saturation ratio is not linearly dependent on the persistence probability in case of the QQW in contrast to the CQW.

Let us now discuss the results for sites $x \neq x_D$; for a site separated by a distance r from x_D the occupation probability can be written as $f(x_D + r, t, x_D, t_R)$. The ratio of the occupation probabilities $f(x_D + r)/f_0(x_D + r)$ (in short hand notation) shows remarkably different behaviour for $r > 0$ and $r < 0$ at a finite value of t (Fig. 6). For $r > 0$, the ratio goes to zero at a finite value of r , the decay is smooth for large values of t_R while for small t_R , the decay is accompanied by small oscillations. For $r < 0$, the ratio shows an irregular behaviour with r , several peaks occur with the peak values much greater than 1. However, looking carefully at Fig. 2, it is evident that for sufficiently large $|r|$, when $r < 0$, the QQW and SIW behave in the same way, and the ratio is not much affected by the removal of the boundary. Hence one can say that the memory effects are strong here.

As function of time, the product of $\frac{f(x_D + r)}{f_0(x_D + r)}$ and $\frac{f(x_D)}{f_0(x_D)}$ are plotted in the inset of Fig. 6 for $x_D = 10$ and $t_R = 50$. The product approaches unity for all r , although for $r > 0$ it reaches unity from below and for $r < 0$ from above. this product can be identified as the ratio of two correlation functions $g(x_D + r, t, x_D, t_R)$ and $g_0(x_D + r, t, x_D, t_R)$ where

$$g(x_D + r, t, x_D, t_R) = f(x_D + r, t, x_D, t_R)f(x_D, t, x_D, t_R) \quad (5)$$

and $g_0(x_D + r, t, x_D, t_R)$ is obtained by replacing f by f_0 in eq.(5). Thus we note that the ratio of the correlation functions approach unity irrespective of the value of r for finite values of t_R .

The ratio of the two correlation functions may also be estimated for the classical case. Using the result of eq.(4),

$\frac{g_c}{g_{c0}}$ is simply the square of the persistence probability and is less than 1 always irrespective of the value of r and t . This is clearly different from the quantum case where the ratio goes to 1 for all r at large t . Moreover, in the quantum case there is a time dependence which is strongly dependent on the sign of r at small t .

IV. SUMMARY AND DISCUSSIONS

In summary, we have studied quenching in a quantum system in a completely different sense compared to earlier works. The observation that the occurrence probability of a QRW may actually be enhanced by quenching is one of the main results of this study. This is a purely quantum mechanical effect. Having the detector up to time t_R means the quenched walker cannot go beyond x_D , and the undetected walker will move away from x_D . However, at a later time $t > t_R$, when the walker is free once again, it can move towards x_D and go beyond. From Fig. 2, it can also be seen that that most of the contributions to x_D and beyond come from the density of walkers closer to it, as the occurrence probability far way from x_D are not much affected with the removal of the detector. At later times after the removal of the detector, the occupation probability profile approaches the IW picture as the local hill like structures smoothen out. This happens closest to x_D at earlier times and slowly the further parts are affected. In comparison, in the infinite walk case, the walker has moved reasonably away from x_D at t_R , such that the ratio can be greater than unity close to x_D . But when t_R is greater than t_R^{lim} , the ratio can no longer exceed unity. At even later times, the ratio saturates as the “memory” of the detector gets erased in time. On the other hand we find that memory effects are strong for $x \ll x_D$ where the removal of the detector is more or less irrelevant. Hence the effect of quenching is rather local.

Other important results are the scaling behaviour of the quantities like t_R^{lim} with x_D and $(f/f_0)_{sat}$ at x_D as a function of x_D or t_R . Although for QRW, the displacement varies linearly with time, we find that the timescale t_R^{lim} varies with x_D in a quadratic manner. The scaling behaviour of $(f/f_0)_{sat}$ is also drastically different from the classical case.

The present work can be extended in many ways like making the probability of detection dependent on time in a different way and slow quenching can be studied when it decays algebraically. One can also, instead of having a detector from time $t = 0$ to $t = t_R$, manipulate both the times at which the detector is placed and subsequently removed. Quantum random walks of correlated particles have been shown to encode information [18] and quantum walk is capable of universal quantum computation [19]. Quenching of quantum random walks may lead to some new features in such contexts, although it is too early to predict exactly how.

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